

# Formal languages

Strings Languages  
Grammars

# Symbols, Alphabet and strings

- A **symbol** is an atomic entity
- An **alphabet** is a finite set of symbols

$$V = \{s_1, s_2, s_3, \dots, s_n\}$$

- A **string** is a sequence of alphabet symbols

$$S = a_1 a_2 a_3 \dots a_k \quad a_i \in V$$

- The **string length** ( $k$ ) is the number of symbols it contains
- The **empty string**  $\epsilon$  does not contain symbols (its length is 0)

# String operations

- Concatenation

$$X = x_1x_2 \dots x_m \quad Y = y_1y_2 \dots y_n$$

$$XY = x_1x_2 \dots x_my_1y_2 \dots y_n$$

$$|XY| = |X| + |Y| = m + n$$

The concatenation is associative but not commutative

- n-th power  $S^n$

- It's the string obtained by the concatenation of n strings equal to S

- $S^1 = S$  e  $S^0 = \varepsilon$

- $|S^n| = n |S|$

$$S = +id \quad S^2 = +id + id \quad S^3 = +id + id + id \dots$$

# Languages

- A **language** is a set of strings defined on a given alphabet  $V$ 
  - by enumeration (... if its is finite)
  - by set rules
  - by a grammar
- The **cardinality** of a language is the number of strings it contains

$V = \{0, 1\}$  *alphabet (dictionary)*

$L_1 = \{\epsilon, 0, 00, 000, 0000, 00000\}$   $|L_1| = 6$

$L_2 = \{1, 01, 001, 0001, 00001, \dots\} = \{0^n 1 | n \geq 1\}$   $|L_2| = \infty$



# Operations on languages

Languages are sets.. then we can define operators from set theory

- Union

$$L_z = L_x \cup L_y = \{z | z \in L_x \vee z \in L_y\}$$

- Intersection

$$L_z = L_x \cap L_y = \{z | z \in L_x \wedge z \in L_y\}$$

- Difference

$$L_z = L_x - L_y = \{z | z \in L_x \wedge z \notin L_y\}$$

Example

$$L_1 = \{0, 00, 000\} \quad L_2 = \{1, 11, 111\}$$

$$L_1 \cup L_2 = \{0, 00, 000, 1, 11, 111\}$$

$$L_1 \cap L_2 = \emptyset$$

# Concatenation and closure

- Concatenation

$$L_{xy} = L_x \cdot L_y = L_x L_y = \{xy | x \in L_x \wedge y \in L_y\}$$

- n-th power  $L^n$

- It's the language obtained by the concatenation of n languages equal to L
- $L^1 = L$  e  $L^0 = \{\epsilon\}$

- Closure  $L^*$

It's the language obtained by the union of all the n-th powers  $L^n$  on L

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

# Closure and linguistic universe

- Positive closure  $L^+$

It is the language obtained by the union of all the  $n$ -th powers of  $L$  for  $n > 0$

$$L^+ = \bigcup_{i=1}^{\infty} L^i$$

- Linguistic universe

Given an alphabet  $V$ , the linguistic universe on  $V$ , referred to as  $V^*$ , is the set of all the strings that can be built with the symbols in  $V$ , including the empty string

- Language

A language is a subset of the linguistic universe

$$L(V) \subseteq V^*$$

# Examples

$$V = \{0,1\}$$
$$L_1 = \{0,01\} \quad L_2 = \{1,11\}$$

- $L_1 L_2 = \{01,011,0111\}$
- $L_1^* = \{\varepsilon, 0, 00, 000, \dots, 01, 0101, 010101, \dots, 001, 0001, \dots\}$
- $L_2^* = \{\varepsilon, 1, 11, 111, 1111, \dots\}$
- $L_2^+ = \{1, 11, 111, 1111, \dots\}$

$V^*$  contains all the strings made up of 0 and 1  
(binary numbers)

# Grammars

- A grammar is defined by a **set of rules** that allow us to **generate** all the strings in a given language
- A grammar  $G$  is a tuple

$$G = \{T, N, P, S\}$$

- $T$  is the alphabet of terminal symbols (appearing in the language strings)
- $N$  is a (finite) set of non terminal symbols (the syntactic categories)
  - $N \cap T = \emptyset$
- $P$  is the set of grammar rules (production rules)
- $S \in N$  is the start symbol (sentence symbol)

# Production rules

- A production rule is a relation between a pair of strings

$$\alpha \rightarrow \beta$$

where  $\alpha \in (T \cup N)^+$  contains at least a symbol in  $N$  and  $\beta \in (T \cup N)^*$

- The production rule states that  $\alpha$  (the left side) can be replaced (rewritten) by  $\beta$  (the right side)

$$T = \{a, b, c\}$$

$$N = \{A, B, C\}$$

$A \rightarrow abc$   
 $A \rightarrow aBbc$   
 $Bb \rightarrow bB$   
 $Bc \rightarrow Cbcc$   
 $bC \rightarrow Cb$   
 $aC \rightarrow aaB$   
 $aC \rightarrow aa$

production rules

$$L(G) = \{a^n b^n c^n \mid n \geq 1\}$$

$$\begin{aligned}
 A &\Rightarrow aBbc \Rightarrow abBc \Rightarrow abCbcc \\
 &\Rightarrow aCbbcc \Rightarrow aabbcc
 \end{aligned}$$

# Derivation

- The generative mechanism defined by the grammar is based on the **derivation operation**: a substring corresponding to the left side of a rule is replaced by the string on the right side

$abBc \Rightarrow abCbcc \Rightarrow aCbbcc \Rightarrow aabbcc$   
 $Bc \rightarrow Cbcc \quad bC \rightarrow Cb \quad aC \rightarrow aa$

- Given two strings  $\gamma = \varphi\alpha\lambda$  e  $v = \varphi\beta\lambda$ , we say that  $\gamma$  **derives in one step**  $v$  ( $\gamma \Rightarrow v$ ) if  $\alpha \rightarrow \beta$  is a production rule in  $G$
- We say that  $\gamma$  **derives**  $v$  ( $\gamma \Rightarrow^* v$ ) **in 0 or more steps** if there exists a sequence of derivations in one step such that

$$\gamma = \alpha_0 \Rightarrow \alpha_1 \Rightarrow \dots \Rightarrow \alpha_{n-1} \Rightarrow \alpha_n = v$$

# The language of G

- Given a grammar G the generated language is the set of strings, made up of terminal symbols, that can be derived in 0 or more steps from the start symbol

$$L_G = \{ x \in T^* \mid S \Rightarrow^* x \}$$

Example: grammar for the arithmetic expressions

$T = \{\text{num}, +, *, (, )\}$

$N = \{E\}$

$S = E$

$E \rightarrow ( E )$   
 $E \rightarrow \text{num}$   
 $E \rightarrow E * E$   
 $E \rightarrow E + E$

start symbol  
 $E \Rightarrow E * E \Rightarrow ( E ) * E \Rightarrow ( E + E ) * E$   
 $\Rightarrow ( \text{num} + E ) * E \Rightarrow (\text{num} + \text{num}) * E$   
 $\Rightarrow (\text{num} + \text{num}) * \text{num}$   
 language phrase



# Classes of generative grammars

- The classification is known as **Chomsky hierarchy** and is based on the structure of the production rules
  - **Type-0 grammars (unrestricted grammars)**
    - The only restriction is that the left side of productions rules must contain at least one non terminal symbol.
    - They generate all the language that can be recognized by a Turing machine
  - **Type-1 grammars (context-sensitive grammars)**
    - The production rules have the structure  $\phi X \gamma \rightarrow \phi \alpha \gamma$  being  $\phi, \gamma \in (T \cup N)^*$ ,  $X \in N$  and  $\alpha \in (T \cup N)^+$
    - The replacement of  $X$  by  $\alpha$  can only take place in the context of  $\phi$  and  $\gamma$ , i.e. when  $X$  appears between the strings  $\phi$  and  $\gamma$
  - **Type-2 grammars (context-free grammars)**
    - The production rules have the structure  $X \rightarrow \alpha$  being  $X \in N$  and  $\alpha \in (T \cup N)^*$
  - **Type-3 grammars ([right] regular grammars)**
    - The production rules have the structure  $X \rightarrow sY$  or  $X \rightarrow s$  being  $X, Y \in N$  and  $s \in T^*$

# Regular languages

- Regular languages can be described by different formal models, that are equivalent
  - Finite State Automata (FSA)
  - Regular grammars (RG)
  - Regular Expressions (RE)
- Each formalism is suited for a given specific task
  - A finite state automaton defines a **recognizer** that can be used to determine if a string belongs to a given regular language
  - Regular grammars define a generative model for the strings in the language
  - Regular expressions describe the structure of the strings in the language (they define a **pattern**)

# Regular expressions – constants & variables

- We define a set of constants and algebraic operators
  - Constants
    - An alphabet symbol
$$s \in V$$
    - The empty string
$$\epsilon \in V^*$$
    - The empty set
$$\emptyset$$
  - Variables
    - A variable represents a “nickname” for a pattern defined by a regular expression

# Regular expressions – Value of a RE

- The value of a regular expression  $E$  corresponds to a language on  $V$ , referred to as  $L(E)$ 
  - If  $E = s$ ,  $s \in V$  then  $L(E) = \{s\}$ 
    - The value of a RE corresponding to a constant symbol is a language containing the only string of length 1 containing the given terminal symbol
  - If  $E = \varepsilon$  then  $L(E) = \{\varepsilon\}$ 
    - The value of a RE corresponding to the empty string is a language containing only the empty string
  - If  $E = \emptyset$  then  $L(E) = \emptyset$ 
    - The value of a RE corresponding to the empty set is a language that contains no elements
  - A variable is associated to the value of the regular expression to which it refers

# Regular expressions- Operators: union

- We define three operators that allow us to combine REs to yield a new RE

- Union of two RE  $U = R \mid S$

- $L(R \mid S) = L(R) \cup L(S)$

$$L(0) = \{0\} \quad L(1) = \{1\} \quad L(0|1) = \{0, 1\}$$

- The operator corresponds to the set union operator and consequently has the following properties
  - Commutativity  $R \mid S = S \mid R$
  - Associativity  $R \mid S \mid U = (R \mid S) \mid U = R \mid (S \mid U)$
- The cardinality of the resulting language is such that

$$L(R|S) \leq |L(R)| + |L(S)|$$

# Regular expressions – Concatenation

## ▪ Concatenation of two RE $C = RS$

- $L(RS) = L(R)L(S)$  – The value of the RE is the language defined by the concatenation of all the strings in  $L(R)$  with those ones in  $L(S)$

$$R = a \quad S = b \quad RS = ab \quad L(RS) = \{ab\}$$

- The operator is not commutative ( $RS \neq SR$  in general)
- The operator is associative  $RSU = (RS)U = R(SU)$
- The cardinality of the language resulting from the concatenation of two regular expressions is such that

$$|L(RS)| \leq |L(R)| \cdot |L(S)|$$

- The same string may be obtained by the concatenation of different strings in  $L(R)$  and  $L(S)$

# Regular expressions- an example

$$R = a \mid (ab) \quad S = c \mid (bc)$$

$$\begin{aligned} RS &= (a \mid (ab))(c \mid (bc)) = ac \mid (ab)c \mid a(bc) \mid (ab)(bc) = \\ &= ac \mid abc \mid abbc \end{aligned}$$

$$L(RS) = \{ac, abc, abbc\}$$

The **distributive property** of concatenation with respect to union holds

$$(R (S \mid T)) = RS \mid RT \quad ((S \mid T) R) = SR \mid TR$$

# Regular expressions–Kleene closure

- The **Kleene closure** is a (suffix) unary operator

$$(R)^*$$

- It has the maximum priority among all operators (use brackets!)
- It represents 0 or more concatenations of the expression  $R$
- $L(R^*)$  contains
  - The empty string  $\varepsilon$  (it corresponds to 0 concatenations of  $R$  –  $R^0$ )
  - All the strings in  $L(R)$ ,  $L(RR)$ ,  $L(RRR)$ ,.... that is

$$L(R^*) = \cup_{i=0}^{\infty} L(R^i)$$

It corresponds to the (improper) regular expression

$$L(R^*) = \varepsilon \mid R \mid RR \mid RRR \mid \dots \mid R^n \mid \dots$$



# Regular expressions – examples & precedence

$$R = (a \mid b) \quad L(R) = \{a, b\}$$

$$R^* = (a \mid b)^* \quad L(R^*) = \{\varepsilon, a, b, aa, ab, bb, ba, aaa, aba, \dots\}$$

- The operator precedence is the following
  - Kleene closure (highest priority)
  - Concatenation
  - Union (lowest priority)
- Parentheses () are needed to write correct (and readable) REs

$$R = a \mid bc^*d = a \mid b(c^*)d = (a) \mid (b(c^*)d) = ((a) \mid (b(c^*)d))$$

$$L(R) = \{a, bd, bcd, bccd, \dots, bc^nd, \dots\}$$

# Regular expressions- examples & variables

- Variables names in a programming language
  - Strings starting with a letter and containing alphanumeric characters

$\text{alpha} = A \mid B \mid C \mid \dots \mid Z \mid a \mid b \mid \dots \mid z$

$\text{numeric} = 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

$\text{variableid} = \text{alpha} (\text{alpha} \mid \text{numeric})^*$

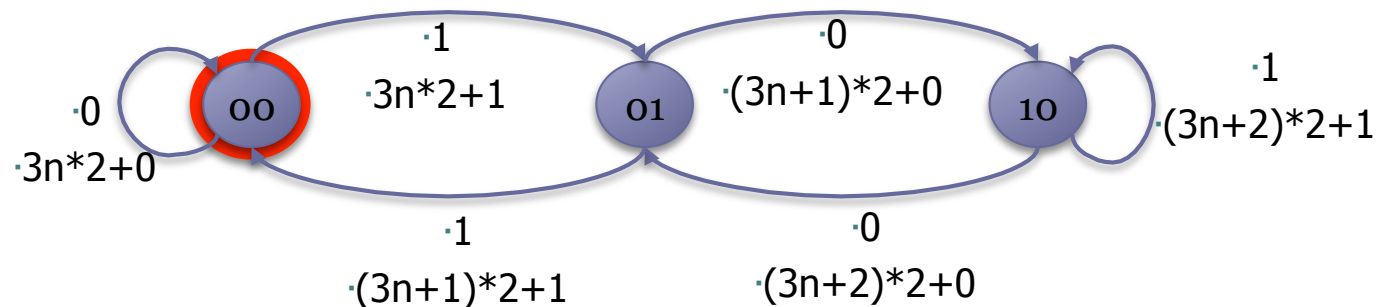
$L(\text{variableid}) = \{A, B, \dots, a, \dots, z, AA, \dots, V1, \dots, i1, \dots, \text{myvar}, \dots\}$

# Regular expressions- examples

- All the strings made up of 0,1 such that
  - they end with 0 -  $R = (0 \mid 1)^*0$
  - they contain at least one 1 -  $R = (0 \mid 1)^*1(0,1)^*$
  - they contain at most one 1 -  $R = 0^*10^*$
  - they have in the third rightmost position a 1
    - $R = (0 \mid 1)^*1(0 \mid 1)(0 \mid 1)$
  - they have even parity (an even number of 1s) -  $R = (0 \mid 10^*1)^*$
  - all the subsequences of 1s have even length -  $R = (0 \mid 11)^*$
  - as binary numbers they are the multiples of 3 (11)
    - $R = (0 \mid 11 \mid 1(01^*0)^*1)^*$

# Regular expressions- multiples of 3 in binary representation

- The regular expression can be derived from the remainder computation for a division by 3
  - The remainders are 00, 01, 10
  - We can build a finite state automaton that computes the remainder when scanning the number from left to right (i.e. by adding a bit at the end at each step)



- The paths from 00 to 00 are  $0^* \mid 11^* \mid 101^*01$  and their concatenations....

# Regular expressions- equivalence

- Two regular expressions are **equivalent** if they define the same language

$$R \equiv S \iff L(R) = L(S)$$

- by exploiting the algebraic equivalences among expressions we can simplify the structure of regular expressions
  - Neutral element
    - union  $(\emptyset \mid R) = (R \mid \emptyset) = R$
    - concatenation  $\varepsilon R = R \varepsilon = R$
  - Null element
    - concatenation  $\emptyset R = R \emptyset = \emptyset$
  - Commutativity (union) and Associativity (union and concatenation)

# Regular expressions- algebraic equivalences

- Distributivity of concatenation with respect to union
  - left -  $R(S|T) = RS | RT$     right -  $(S|T)R = SR | TR$
- Union idempotence
  - $(R | R) = R$
- Equivalences for Kleene closure
  - $\emptyset^* = \varepsilon$
  - $RR^* = R^*R = R^+$  (one or more concatenations of strings in  $L(R)$ )
  - $RR^*|\varepsilon = R^*$
- Example

$$(0|1)^*(10|11)(0|1) = (0|1)^*1(0|1)(0|1) = (0|1)^*1(00|01|10|11) = (0|1)^*(100|101|110|111)$$

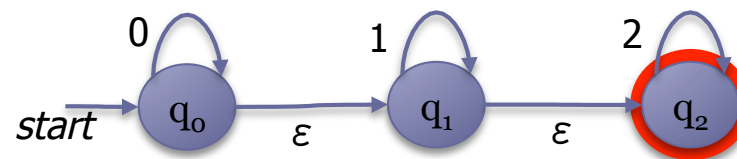
# Regular expressions and FSA

- It is possible to transform a RE  $R$  into a non-deterministic finite state automaton that recognizes the strings in the language defined by  $R$
- It is possible to transform a (non-deterministic) finite state automaton into a RE that defines the language recognized by that automaton

RE and FSA (NFSA) are equivalent models for the definition of regular languages

# FSA with $\epsilon$ -transitions

- This model extends the class of finite state automata by allowing state transitions labeled by the empty-string symbol  $\epsilon$  ( $\epsilon$ -transitions)
  - The consequence is that the automaton can perform a state transition even without reading a symbol from the input string
  - The automaton accept the input string if there exists at least one path  $w$  from the start state to a final accepting state
    - The path can contain arcs corresponding to  $\epsilon$ -transitions beside those labeled by the symbols in the input sequence
    - The automaton is said to be non-deterministic since more than one path (state sequence) may exist for a given input string



002  $\Rightarrow$  0 0  $\epsilon$   $\epsilon$  2  
 $q_0 q_0 q_1 q_2 q_2$

$$R = 0^*1^*2^*$$

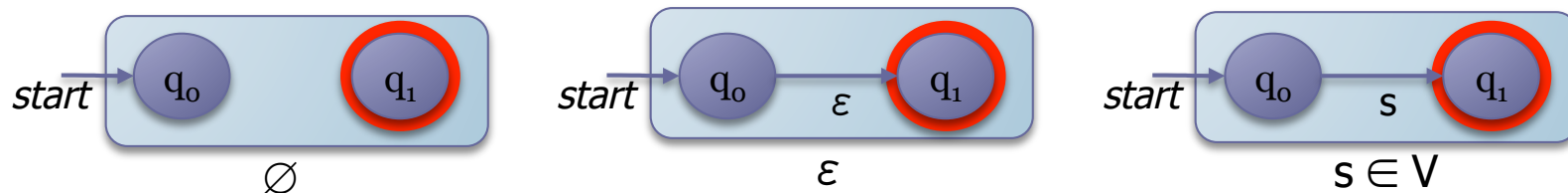


# FSA with $\epsilon$ -transitions - definition

- A **finite state automaton with  $\epsilon$ -transition** is defined by a tuple  $(Q, V, \delta, q_0, F)$  where
  - $Q = \{q_0, \dots, q_n\}$  is the finite set of states
  - $V = \{s_1, s_2, \dots, s_k\}$  is the input alphabet
  - $\delta: Q \times (V \cup \{\epsilon\}) \rightarrow 2^Q$  is the state transition function
    - the actual transition is in general to a set of future states given the presence of  $\epsilon$ -transitions
  - $q_0 \in Q$  is the start state
  - $F \subseteq Q$  is the set of the final accepting states

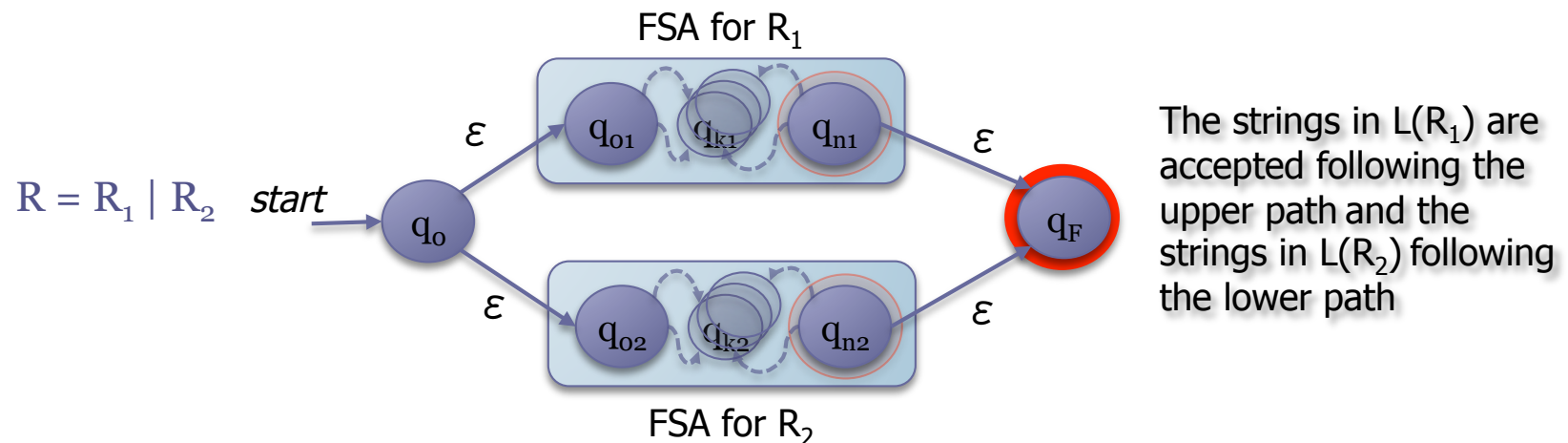
# From a RE to a FSA with $\varepsilon$ -transitions

- Given a RE  $R$  there exists a finite state automaton with  $\varepsilon$ -transitions  $A$  that accepts only the strings in  $L(R)$ 
  - $A$  has only one accepting state
  - $A$  has no transitions to the start state
  - $A$  has no transitions going out of the accepting state<sup>1</sup>
- The proposition can be proved by induction on the number  $n$  of operators in the regular expression  $R$ 
  - $n=0$   
 $R$  has only a constant  $\emptyset, \varepsilon$  or  $s \in V$

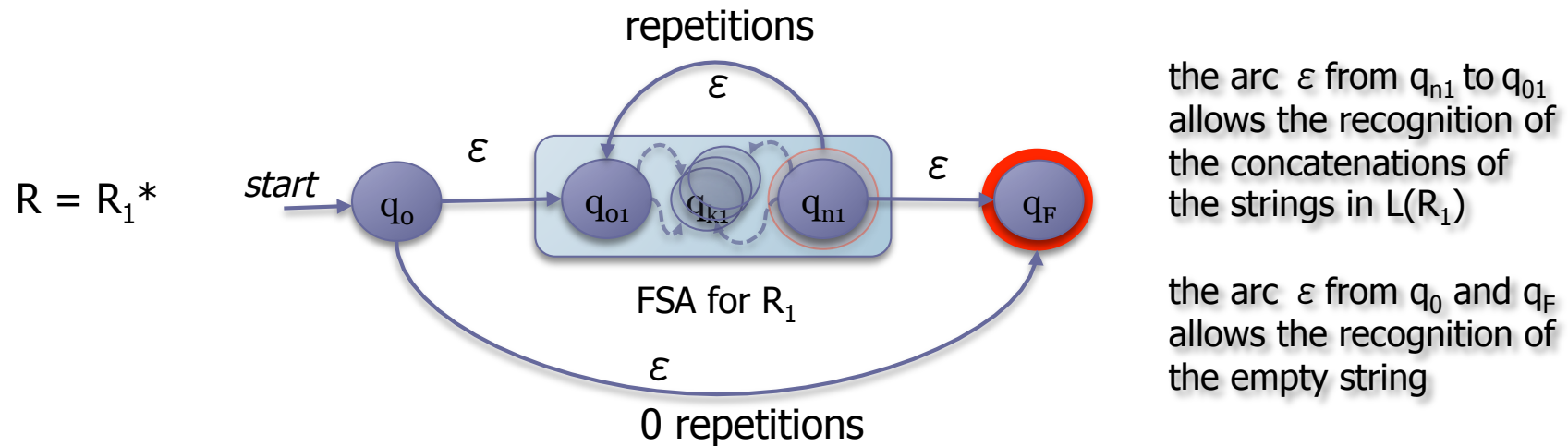
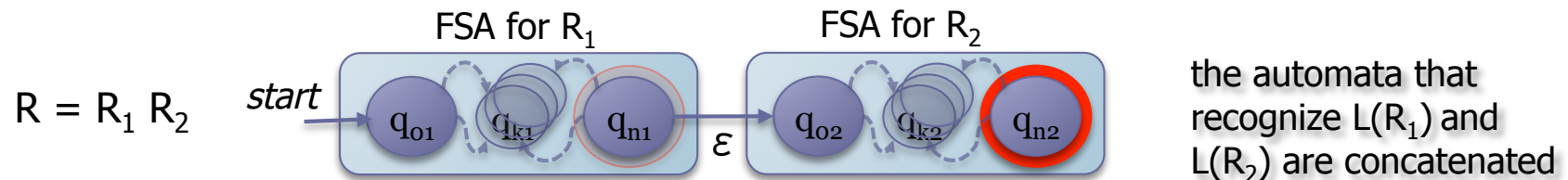


# From a RE to a FSA with $\varepsilon$ -transitions - $n > 0$

- By induction we suppose to know how to construct the equivalent automaton for a RE having  $n-1$  operators
  - One of the defined operators can be added to obtain a RE with  $n$  operators
    - $R = R_1 \mid R_2$
    - $R = R_1 R_2$
    - $R = R_1^*$
 where  $R_1$  and/or  $R_2$  have at most  $n-1$  operators



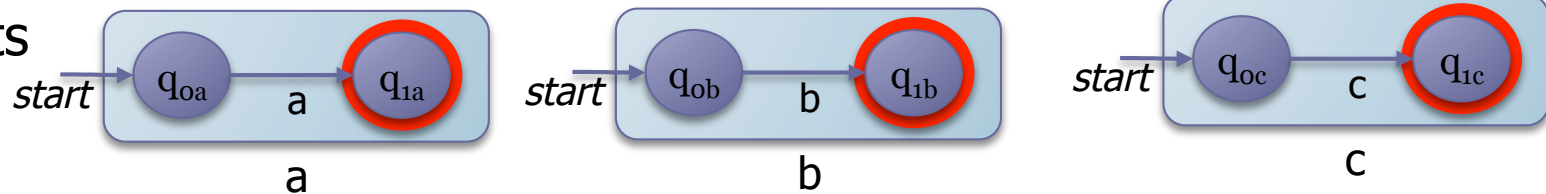
# From a RE to a FSA with $\epsilon$ -transitions - $n > 0$



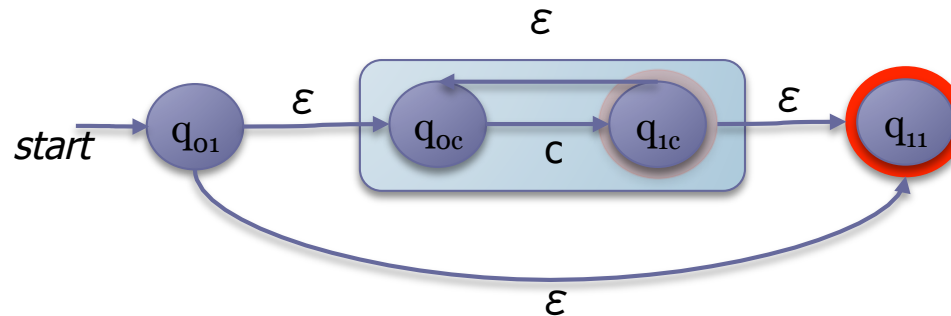
# From a RE to a FSA - example [1]

$$R = a \mid bc^*$$

Constants

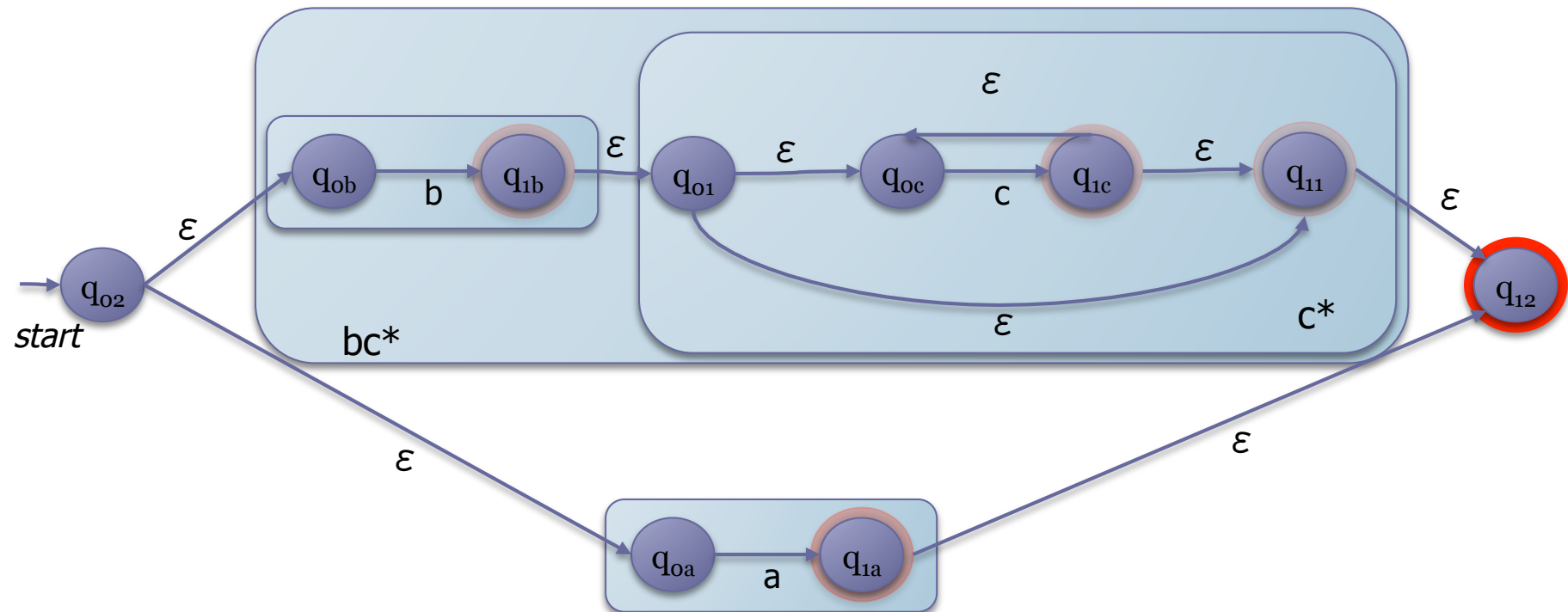


$c^*$



# From a RE to a FSA - example [2]

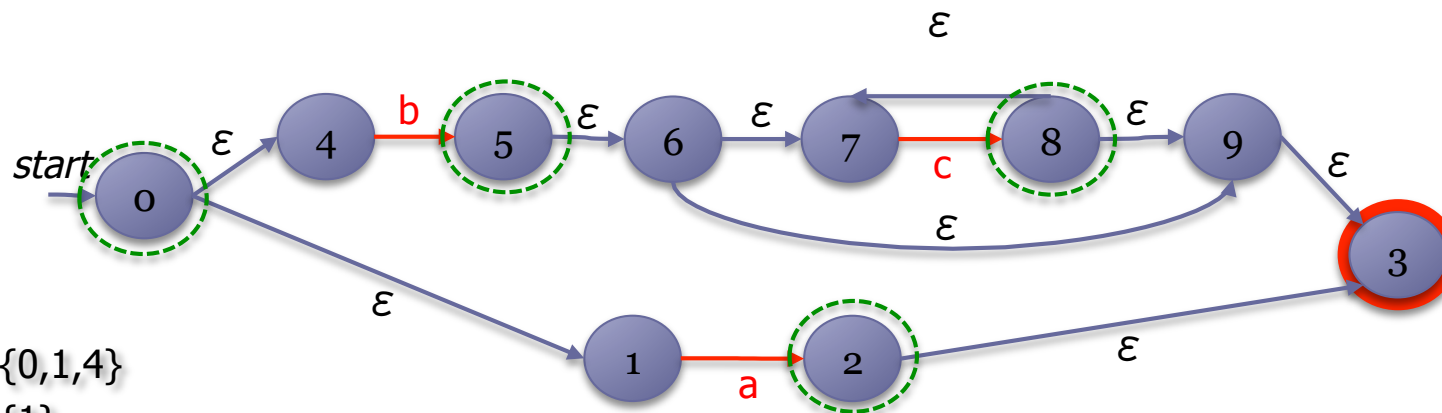
$a \mid bc^*$



# Removal of $\epsilon$ -transitions

- It is always possible to transform an automaton with  $\epsilon$ -transitions into a deterministic automaton (DFSA)
  - if  $q$  is the current state, the automaton may perform any transition to all the states reachable from  $q$  with  $\epsilon$ -transitions
    - it is like the automaton is in state  $q$  and in all its  $\epsilon$ -reachable states
  - For each state  $q$  we need to find all the states that are reachable by  $\epsilon$ -transitions
    - it is a node reachability problem on a graph
    - All the transitions not labeled with  $\epsilon$  are removed
    - A depth-first visit of the graph is performed from any node
  - All the states that are  $\epsilon$ -reachable from  $q$  are associated to the original state  $q$ 
    - these sets represent the candidate states for the DFSA

# From $\epsilon$ -FSA to DFSA - key states



$R(0) = \{0,1,4\}$

$R(1) = \{1\}$

$R(2) = \{2,3\}$

$R(3) = \{3\}$

$R(4) = \{4\}$

$R(5) = \{3,5,6,7,9\}$

$R(6) = \{3,6,7,9\}$

$R(7) = \{7\}$

$R(8) = \{3,7,8,9\}$

$R(9) = \{3,9\}$

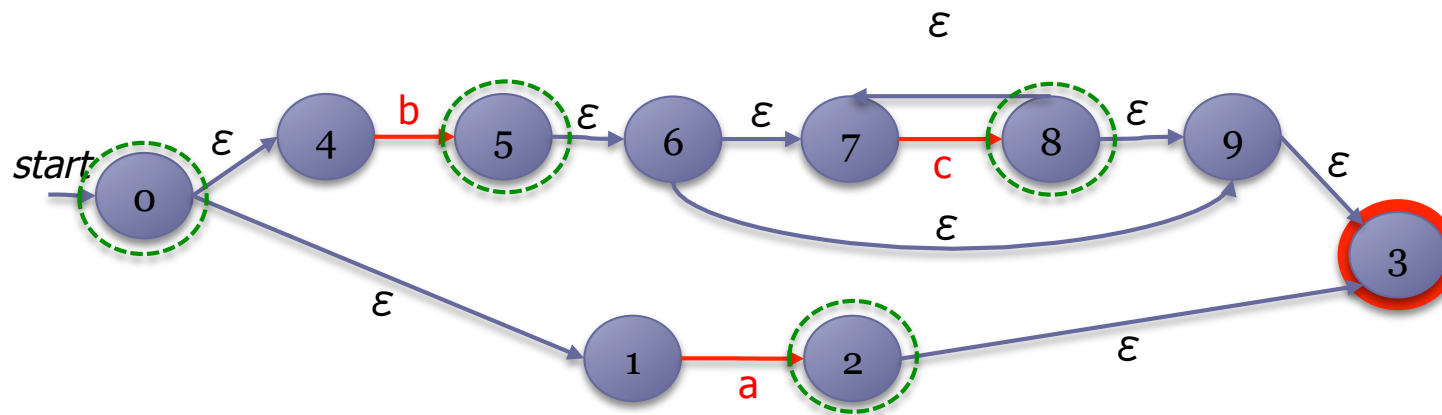
we define the **key states**

- states having an incoming arc labeled with a symbol
- start state

$\cdot SI = \{0,2,5,8\}$



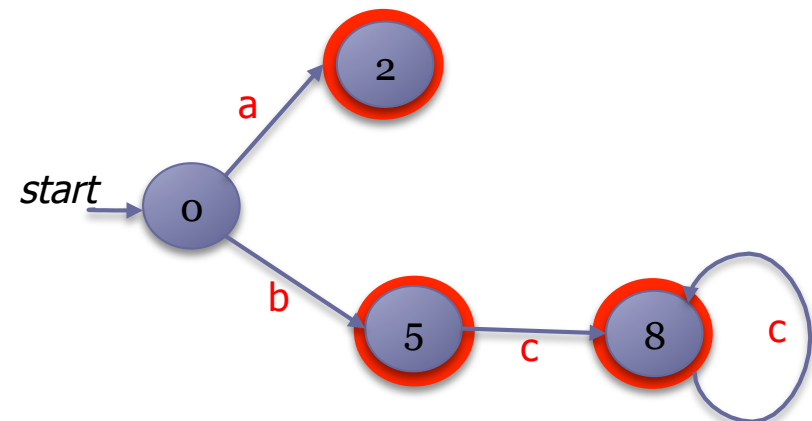
# From $\epsilon$ -FSA to DFSA - transitions



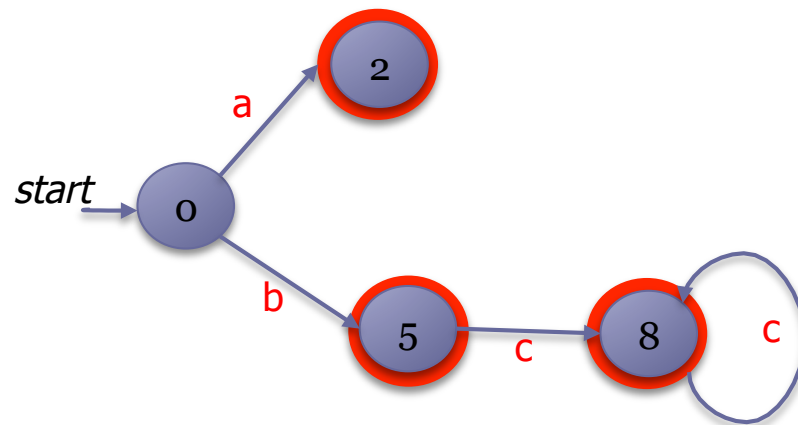
There is a transition from the key state  $i$  to the key state  $j$  labeled with symbol  $s$ , if

- there exists a state  $k$  in  $R(i)$
- there exists a transition from  $k$  to  $j$  with label  $s$

A key state  $i$  is accepting if at least one accepting state is in  $R(i)$

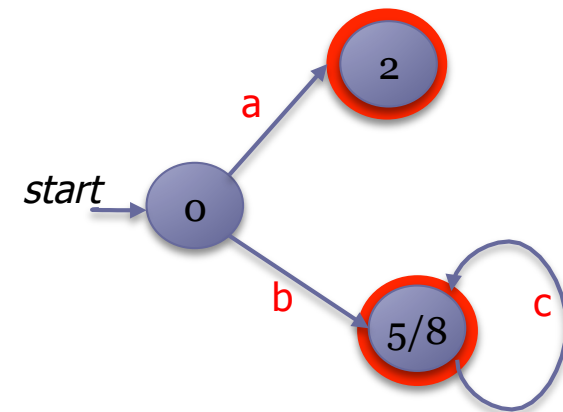


# From $\varepsilon$ -FSA to DFSA – NDFSA & minimization



The resulting automaton may be non deterministic

- it may have more than one transition going out of the same state with the same symbol
- there is an algorithm to transform this type of NDFSA to an equivalent DFSA (we add a state for each set of states that are reachable with the same symbol)
- NDFSA, DFSA and  $\varepsilon$ -FSA are equivalent models

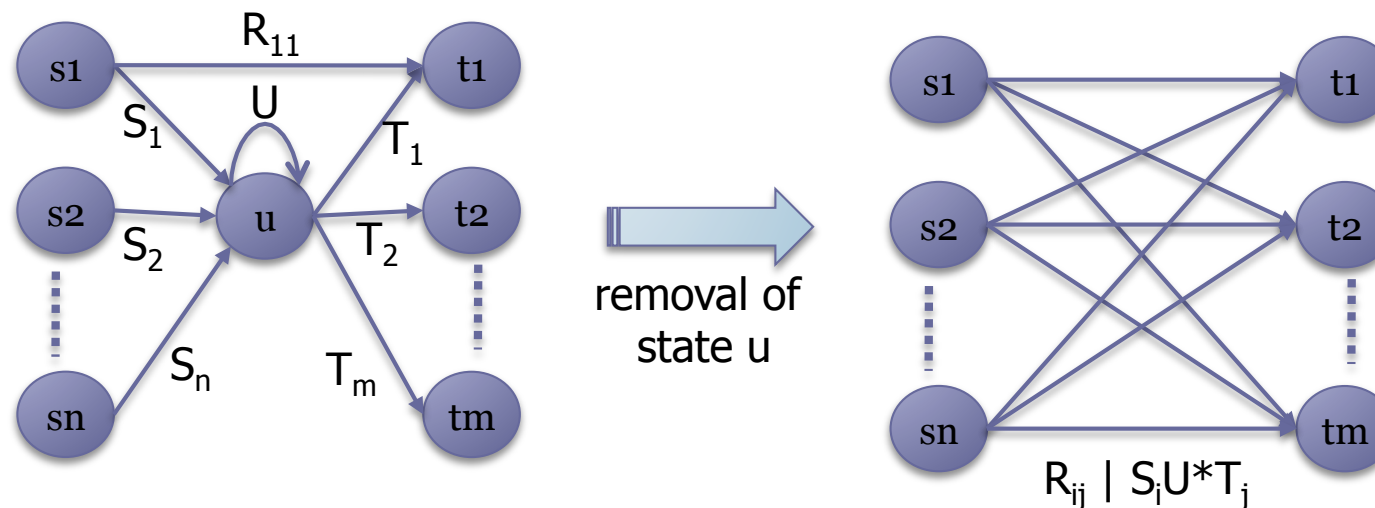


The automaton may be minimized by finding the classes of equivalent states

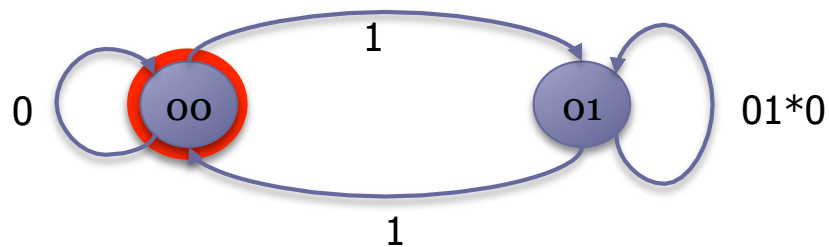
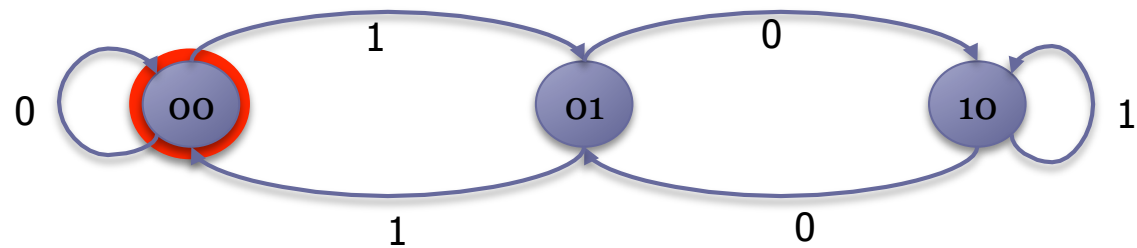
- equivalence at 0 steps (same output)  
 $\{0\}$   $\{2,5,8\}$
- equivalence at 1 step (input a b c)  
 $\{0\}$   $\{2\}$   $\{5,8\}$  (they differ for c)
- from 2 step 5 and 8 are indistinguishable

# From FSAs to REs

- For any FSA  $A$  there exists a regular expression  $R(A)$  that defines the same language (set of strings) recognized by  $A$ 
  - It can be obtained by a progressive removal of states
  - The arcs are labeled by regular expressions that describe the paths passing through the set of states removed up to a give step

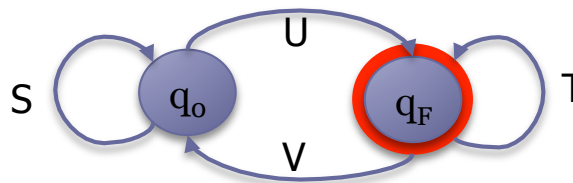


# From FSAs to REs - example



# From FSAs to REs – complete reduction

- The reduction process must be repeated for each accepting state
  - The final regular expression is the union of the regular expressions obtained for each accepting state
  - If we consider an accepting state, the corresponding regular expression is the label of the path from the start state  $q_0$  and the accepting state  $q_F$ 
    - All the state are removed by the reduction process until only  $q_0$  and  $q_F$  are left



- We consider the regular expression describing the paths that originate in  $q_0$  and end into  $q_F$

$$R = S^*U(T|VS^*U)^*$$

# Applications & standard for REs

- There are several software applications/libraries that exploit Res or support the management of REs
  - search commands in text editors
  - programs to search patterns in files (grep, awk)
  - library functions/procedures that implement regular expression matching (regex in stdlib C, RE support in PHP, perl, ecc.)
  - programs to generate lexical scanners (lex)
- The IEEE POSIX 1003.2 standard defines a syntax/semantics to implement Res
  - two levels: Extended RE and Basic RE (obsolete)

# POSIX 1003.2 - operators

- The union operator is represented by the character |
- The concatenation operator is implicitly obtained by writing the sequence of symbols or symbol classes to be concatenated (or REs)
- The standard defines also following unary operators
  - \* - 0 or more occurrences of the operand on the left
  - + - 1 or more occurrences of the operand on the left
  - ? - 0 or 1 occurrence of the operand on the left
  - {n} - exactly n occurrences of the operand on the left
  - {n,m} - between n and m occurrences of the operand on the left
  - {n,} - more than n occurrences of the operand on the left

# POSIX 1003.2 – constants 1

- Constants/atoms (operands for unary/binary operators)
  - A character
    - Special characters are represented by an escape sequence \  
e.g. \\ \| \. \^ \\$
  - A RE between ()
  - The empty string ()
  - A square bracket expression [ ] – a character class
    - [abcd] any of the listed characters
    - [0-9] the digits between 0 and 9
    - [a-zA-Z] the lowercase and uppercase charactersto specify the character “minus” – it should be listed in the first position
  - Any character .
  - The start of a line ^
  - The end of a line \$



# POSIX 1003.2 – constants 2

- Exclusion of a character class
  - `[^abc]` all characters excluding a b c (the character ^ strictly follows [ )
- Predefined character classes
  - `[:digit:]` only digits between 0 and 9
  - `[:alnum:]` any alphanumeric character between 0 and 9, a and z or A and Z
  - `[:alpha:]` any alphabetical character
  - `[:blanc:]` space and TAB
  - `[:punct:]` any punctuation character
  - `[:upper:]` any uppercase alphabetical character
  - `[:lower:]` any lowercase alphabetical character
  - etc...

# POSIX 1003.2 - examples

- A RE matches the first substring having maximum length in the input text that verifies the specified pattern
- Examples
  - strings containing the vowels in alphabetical order  
`.*a.*e.*i.*o.*u.*`
  - numbers with decimal digits  
`[0-9]+\.[0-9]*|\.[0-9]+`
  - number with two decimal digits  
`[0-9]+\.[0-9]{2}`

# Lexical analysis & lex

- The **lex** command is used to generate a **scanner** that is a software module/program that is able to recognize lexical entities in a text
  - The scanner behavior is described in a source lex file that contains the scanning rules (the patterns) and the associated programming code (C)
  - lex generates a source program (C) `lex.yy.c` that implements the function `yylex()`
  - The source is compiled and linked with the lex library (`-lfl`)
  - The executable scans an input file searching for the matches of the regular expressions (patterns)
    - When a RE matches a substring in the input, the associated code (C) is executed

# Scanner usage

- The generated scanner allows us to split the input file into tokens (atomic substrings) such as
  - identifiers
  - constants
  - operators
  - keywords
- Each token is defined by a RE
- The target language for flex is C, but there exists also similar applications to generate code in other high-level programming languages (e.g. Jflex for Java)

# Flex - Fast Lexical Analyzer Generator

- It is used for the generation of scanners
  - By default the text that does not match any rule is copied to the output, otherwise the code associated to the matching RE is executed
  - The rule file has the following structure

definitions

%%

rules

%%

C code

definitions of names  
start conditions

pattern (RE) ACTION

C code (optional) that is directly  
copied to lex.yy.c  
- code of utilities (e.g. functions)

# Flex – definitions of names

- It's a directive having the following structure

## NAME DEFINITION

- NAME is a identifier starting with a letter
- The definition is referred to as {NAME}
- DEFINITION is a RE

### Example

```
ID [A-Za-z][A-Za-z0-9]*
```

```
defines {ID}
```

- In the “definitions” section the indented lines or lines between %{ and %} (at the beginning of the line) are copied to lex.yy.c

# Flex - Rules

- In the “rules” section text that is indented or delimited by `%{` and `%` at the beginning of the section can be used to declare variable local to the scan procedure (inside its scope)
- A rule has the following structure

PATTERN (RE) ACTION

- PATTERN is a RE with the following additions
  - It is possible to specify strings between “ ” where special characters (e.g. `[]`) are not interpreted as operators
  - It is possible to specify characters by their hexadecimal code (e.g. `\x32`)
  - `r/s` matches `r` only if it is followed by `s`
  - `<s1,s2,s3>r` matches `r` only if the scanner is in one of the conditions `s1,s2,s3..` (`<*>` can be used to specify any condition)
  - `<<EOF>>` is matched by the end-of-file

# Flex – rule matching

- The input text is progressively scanned from the beginning
- If more rules are satisfied at the current character, the rule matching the longest substring is activated
- If more than one rule matches the same substring, the rule that is defined first is applied
  - once a match is found, the matching token is available in the `ytext` variable; the variable `yylen` stores the length of the matching substring
  - a match causes the execution of the associated action
  - if there is no match the input is copied to the output by default



# Flex - example: line/word/character counter

```

global variables    int nLines = 0, nChars=0, nWords = 0;
                    %%
PATTERN (RE)        \n                                ++nLines; ++nChars;
                    [^[:space:]]+                    ++nWords; nChars += yyleng;
                    .                                ++nChars;
                    %%
                    ACTION
main()
{
scanner call        yylex();
                    printf("%d lines, %d words, %d characters\n",
                           nLines, nWords, nChars);
}

```

If the rules for the single characters `.` and for the words `[^[:space:]]+` are inverted in the list the scanner does not work correctly (the first rule always matches)

The scanner reads its input from the stream `yyin` (by default `stdin`)

# Flex – example: minimal programming language

NAME  
definitions

```
%{
#include <math.h>
}%


DIGIT [0-9]
ID    [a-zA-Z][a-zA-Z0-9]*

%%

{DIGIT}+ {printf("int %s (%d)\n", yytext, atoi(yytext));}
{DIGIT}+"."{DIGIT}+ {printf("float %s (%f)\n", yytext, atof(yytext));}
if|for|while|do {printf("keyword %s \n", yytext);}
int|float|double|struct {printf("data type %s \n", yytext);}
{ID} {printf("identifier %s \n", yytext);}
"+"|"-"|"*"|"/" {printf("arithmetic operator %s \n", yytext);}
"//[^\n]*" /* removes comments on one line */
"/*"(.|\n)*"*/" /* removes comments on multiple lines */
"{ "|" "}" {printf("block delimiter \n");}
[ \t\n]+ /* removes spaces etc*/
. {printf("invalid char %s \n", yytext);}

%%
```

yytext is the matching string



# Flex – variables and actions

- Two variables are used to reference the substring matching the RE
  - `yytext`
    - by default is `char *` being a reference to the memory buffer where the original text is stored
    - using the command `%array` in the first section of the lex source file, we may force the variable to be a `char []`, i.e. a copy of the original buffer (it can be rewritten without the risk of affecting the scanner behavior)
  - `yyleng`
    - it is the character length of the substring matching the RE
- The action is used to specify the (C) code to be executed when the RE is matched by a substring in the input text
  - the action is written C (using `{}` if it spans more than one line)
  - the execution of a return statement causes the exit from the `yylex()` function. If `yylex()` is called again thereafter the scan restarts from the input position where it was stopped.

# Flex – special directives in actions

- Special directives can be specified in the action code (they are C macros)
  - **ECHO**
    - copies ytext to output
  - **BEGIN(condition)**
    - activates the scanner state named “condition”. The scanner states allow the selective activation of subset of rules.
  - **REJECT**
    - Activates the second best matching rule (it may be verified by the same string or by a prefix)

```
\n          ++nLines; ++nChars;  
pippo      ++nPippo; REJECT;  
[^[:space:]]+ ++nWords; nChars += yyleng;  
.  
          ++nChars;
```

# Flex – library functions

- Scanner library functions can be used in the actions
  - `yymove()`
    - the following match is searched and its value is added to `yytext`
  - `yyless(n)`
    - `n` characters are pushed back into the input buffer
  - `unput(c)`
    - the character `c` is pushed back into the input buffer
  - `input()`
    - the next character is read moving forward by 1 the position of the read cursor
  - `yyterminate()`
    - it is equivalent to the return statement
  - `yyrestart()`
    - resets the scanner to read a new file (it does not reset the current condition) – `yyin` is the file used for reading (stdin by default)

# Flex - conditions

- The **conditions** allows a selective activation of rules

`<SC>RE {action;}`

- the rule is activated only if the scanner is in condition SC
- the conditions are defined in the initialization section of the lex source
  - %s SC1 SC2 – inclusive conditions (the REs without any condition are active)
  - %x XC1 XC2 – exclusive conditions (only those REs with the current condition are active – a scanner “local to the current condition” is selected)
- the scanner enters into condition SC after the execution of the command
  - BEGIN(SC)
- the initial condition is entered with the command
  - BEGIN(o) or BEGIN(INITIAL)
- YYSTART stores the current state (it is a int variable)
- the REs active in the same condition can be declared in a block `<SC>{...}`

# Flex - conditions: example

RE in condition  
COMMENT

```
%x COMMENT
int nCLines=0;
%%
"/*" BEGIN(COMMENT); nCLines++;
<COMMENT>[^*\n]*      /* skips the character not * and \n */
<COMMENT>"*" + [^*/\n]* /* skips * not followed by * or */
<COMMENT>\n           nCLines++;
<COMMENT>"*" + "/"     BEGIN(INITIAL);
[^/]*|"/" [^*/]*      /* skips characters outside comments */
%%
```

- Counts the comment lines in C (`/* ..... */`)

# Flex – example: parsing of string constants in C

```

%x string
%%
char str_buf[1024], *str_buf_ptr;
string start "
    \" str_buf_ptr = str_buf; BEGIN(string);
<string> {
    \" { BEGIN(INITIAL); *str_buf_ptr = '\\0';
        printf(\"%s\\n\",str_buf); }
    \\n printf(\"String is not terminated correctly\\n\"); yyterminate();
    \\[0-7]{1,3} {int r; sscanf(yytext+1,\"%o",&r);
        if(r>0xff) {printf(\"ASCII code is not valid\\n\"); yyterminate();}
        *(str_buf_ptr++) = r; }
    \\[0-9]+ printf(\"octal code is not valid\\n\"); yyterminate();
    \\n      *(str_buf_ptr++) = '\\n';
    ...
    \\(.|\\n)  *(str_buf_ptr++) = yytext[1];
    [^\\n\\\"']+ {int i; for(i=0;i<yytext[i];) *(str_buf_ptr++) = yytext[i];}
}
%%

```

string start "

string parsing up to "



# Flex – multiple input buffers

- The possibility to use multiple input buffers supports the “concurrent” scanning of more than one file (e.g. include)
  - the scan of a file is momentarily interrupted to start the scan of another included file
  - more than one input buffer can be allocated
    - `YY_BUFFER_STATE yy_create_buffer(FILE *file, in size)`
  - the buffer used by the scanner can be selected
    - `void yy_switch_to_buffer(YY_BUFFER_STATE new_buffer)`
    - the scan continues with the new buffer without changes in the scanner condition
  - created buffers can be deallocated
    - `void yy_delete_buffer(YY_BUFFER_STATE buffer)`
  - `YY_CURRENT_BUFFER` references the current buffer
  - the rule `<<EOF>>` allows us to manage the end of the scanning of a file

# Flex – esempio di include

include starts

```
"#include" BEGIN(incl);
```

extract the  
include file  
name and file  
open

```
<incl>{
  [[:space:]]* /* skip spaces*/
  \"[[:alnum:]].]+\" { if(stack_ptr>=MAX_DEPTH) { /*too many nested includes*/ }
                    include_stack[stack_ptr++]=YY_CURRENT_BUFFER;
                    strncpy(filename,yytext+1,yytext-1);
                    filename[yytext-2]='\0';
                    if(!(yyin=fopen(filename,\"r\"))) { /* file open error */ }
                    yy_switch_to_buffer(yy_create_buffer(yyin,YY_BUF_SIZE));
                    BEGIN(INITIAL); }
  [^\[:space:]]+ { /* include error*/ }
}
```

end of included  
file

```
<<EOF>> { if(--stack_ptr<0)
           yyterminate();
          else {
            fclose(YY_CURRENT_BUFFER->yy_input_file);
            yy_delete_buffer(YY_CURRENT_BUFFER);
            yy_switch_to_buffer(include_stack[stack_ptr]) }
          }
```